Optical activity of noncentrosymmetric metals

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We describe the phenomenon of optical activity of noncentrosymmetric metals in their normal and superconducting states. The found conductivity tensor contains the linear in wave vector off diagonal part responsible for the natural optical activity. Its value is expressed through the ratio of light frequency to the band splitting due to the spin-orbit interaction. The Kerr rotation of polarization of light reflected from the metal surface is calculated.

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I. INTRODUCTION

The metals without inversion symmetry have recently become a subject of considerable interest mostly due to the discovery of superconductivity in $CePt_3Si$.[1] Now the list of noncentrosymmetric superconductors has grown to include UIr[2], $CeRhSi_3$[3], $CeIrSi_3$[4], Y_2C_3[5], $Li_2(Pd_{1-x},Pt_x)_3B$[6], KOs_2O_6[7], and other compounds. The spin-orbit coupling of electrons in noncentrosymmetric crystal lifts the spin degeneracy of the electron energy band causing a noticeable band splitting. The Fermi surface splitting can be observed by the de Haas-van Alphen effect discussed theoretically in the paper Ref. 8. The band splitting reveals itself in the large residual value of the spin susceptibility of noncentrosymmetric superconductors at zero temperature.[9] It also makes possible the existence of nonuniform superconducting states that can be traced to the Lifshitz invariants in the free energy.[10] Another significant manifestation of the band splitting is the natural optical activity.

The natural optical activity or natural gyrotropy is well known phenomenon typical for the bodies having no centre of symmetry[11]. The optical properties of a naturally active body resemble those of the magneto-active media having no time reversal symmetry. It exhibits double circular refraction, the Faraday and the Kerr effects. In this case, the tensor of dielectric permeability has linear terms in the expansion in powers of wave vector

$$\varepsilon_{ij}(\omega, \mathbf{q}) = \varepsilon_{ij}(\omega, 0) + i\gamma_{ijl}(\omega) q_l, \tag{1}$$

where $\gamma_{ikl}(\omega)$ is an antisymmetric third rank tensor called the tensor of gyrotropy.

The spacial dispersion term in permeability has been derived by Arfi and Gor'kov[12] in frame of model where "a conductor lacking a center of inversion is simulated by an ordered arrangement of impurities whose scattering potential is asymmetric" (see also[13]). We shall be interested in gyrotropy properties of a clean noncentrosymmetric metal. In metals, it is more natural to describe optical properties in terms of spacial dispersion of conductivity tensor having the following form:

$$\sigma_{ij}(\omega, \mathbf{q}) = \sigma_{ij}(\omega, 0) - i\lambda_{ijl}(\omega) q_l. \tag{2}$$

Here, the gyrotropy tensor $\lambda_{ijl}(\omega)$ is an odd function of frequency.

The gyrotropic tensor has the most simple form in the metals with cubic symmetry. In this case, the Drude part of the conductivity tensor is isotropic $\sigma_{ij}(\omega, 0) = \sigma(\omega)\delta_{ij}$ and the gyrotropic conductivity tensor $\lambda_{ikl}(\omega) = \lambda(\omega) e_{ikl}$ is determined by the single complex function $\lambda(\omega) = \lambda'(\omega) + i\lambda''(\omega)$ such that a normal state density of current is

$$\mathbf{j} = \frac{\epsilon}{4\pi}\frac{\partial \mathbf{E}}{\partial t} + \sigma\mathbf{E} + \lambda \mathrm{rot}\mathbf{E}. \tag{3}$$

In time representation λ is an operator being an odd function of operation of time derivative $\partial/\partial t$.

In the superconducting state besides $\lambda\mathrm{rot}\mathbf{E}$ the gyrotropic part of current density contains also an additional term proportional to the magnetic induction $\mathbf{B}$

$$\mathbf{j}_g = \lambda \mathrm{rot}\mathbf{E} + \nu\mathbf{B}. \tag{4}$$

Here $\nu = \nu(T)$ is a constant coefficient being equal to zero in the normal state.[14,15]

In this paper we present the derivation of current response to the electro-magnetic field with finite frequency and wave vector valid for the normal and the superconducting state of noncentrosymmetric metals with cubic symmetry. We find that the gyrotropy conductivity λ is directly proportional to the ratio of the light frequency to the energy of the band splitting. Then making use the Maxwell equations and corresponding boundary conditions at the surface of noncentrosymmetric metal derived in the Section V we calculate the Kerr rotation of polarization of light reflected from the surface of metal.

II. CURRENT RESPONSE TO ELECTRO-MAGNETIC FIELD

The current response of a clean metal to the electromagnetic field at finite $\mathbf{q}$ and ω can be written following the textbook procedure[16]. In application to our situation one has to remember that due to spin-orbital coupling determined by the dot product of the Pauli matrix vector $\boldsymbol{\sigma}$ and pseudovector $\boldsymbol{\gamma}(\mathbf{k})$, which is odd in respect to

momentum $\boldsymbol{\gamma}(-\mathbf{k}) = -\boldsymbol{\gamma}(\mathbf{k})$ and specific for each non-centrosymmetric crystal structure[9,17], all the values such as single electron energy

$$\xi_{\alpha\beta}(\mathbf{k}) = (\varepsilon(\mathbf{k}) - \mu)\delta_{\alpha\beta} + \boldsymbol{\gamma}(\mathbf{k})\boldsymbol{\sigma}_{\alpha\beta}, \tag{5}$$

velocity $\mathbf{v}_{\alpha\beta}(\mathbf{k}) = \partial\xi_{\alpha\beta}(\mathbf{k})/\partial\mathbf{k}$, the inverse effective mass $(m_{ij}^{-1})_{\alpha\beta} = \partial^2\xi_{\alpha\beta}(\mathbf{k})/\partial k_i\partial k_j$, the Green functions $G_{\alpha\beta}(\tau_1,\mathbf{k};\tau_2,\mathbf{k}') = -\langle T_\tau a_{\mathbf{k}\alpha}(\tau_1)a^\dagger_{\mathbf{k}'\beta}(\tau_2)\rangle$ and $F_{\alpha\beta}(\tau_1,\mathbf{k};\tau_2,\mathbf{k}') = \langle T_\tau a_{\mathbf{k}\alpha}(\tau_1)a_{-\mathbf{k}'\beta}(\tau_2)\rangle$ are matrices in the spin space. Taking this in mind, we obtain

$$\begin{aligned} j_i(\omega_n,\mathbf{q}) = -\frac{e^2}{c}Tr\,\Big[\hat{m}_{ij}^{-1}\hat{n}_e\, + \\ \int\frac{d^3k}{(2\pi\hbar)^3}\mathrm{T}\sum_{m=-\infty}^{\infty}\{\hat{v}_i(\mathbf{k})\hat{G}^{(0)}(K_+)\hat{v}_j(\mathbf{k})\hat{G}^{(0)}(K_-) \\ +\hat{v}_i(\mathbf{k})\hat{F}^{(0)}(K_+)\hat{v}^t_j(-\mathbf{k})\hat{F}^{+(0)}(K_-)\}\Big]\,A_j(\omega_n,\mathbf{q}). \end{aligned} \tag{6}$$

The transposed matrix of velocity is determined as $\hat{\mathbf{v}}^t(-\mathbf{k}) = \partial\hat{\xi}^t(-\mathbf{k})/\partial\mathbf{k}$. The arguments of the zero field Green functions are denoted as $K_\pm = (\Omega_m \pm \omega_n/2, \mathbf{k} \pm \mathbf{q}/2)$. The Matsubara frequencies take the values $\Omega_m = \pi(2m+1-n)T$ and $\omega_n = 2\pi nT$.

One can pass from the spin to the band representation, where the one-particle Hamiltonian

$$H_0 = \sum_{\mathbf{k}}\xi_{\alpha\beta}(\mathbf{k})a^\dagger_{\mathbf{k}\alpha}a_{\mathbf{k}\beta} = \sum_{\mathbf{k},\lambda=\pm}\xi_\lambda(\mathbf{k})c^\dagger_{\mathbf{k}\lambda}c_{\mathbf{k}\lambda} \tag{7}$$

has diagonal form. Here, the band energies are

$$\xi_\lambda(\mathbf{k}) = \varepsilon(\mathbf{k}) - \mu + \lambda|\boldsymbol{\gamma}(\mathbf{k})|, \tag{8}$$

such that two Fermi surfaces are determined by equations $\xi_\lambda(\mathbf{k}) = 0$. The difference of the band energies $2|\boldsymbol{\gamma}(\mathbf{k}_F)|$ characterizes the intensity of the spin-orbital coupling. The Fermi momentum with $\boldsymbol{\gamma} = 0$ is determined by the equation $\varepsilon(\mathbf{k}_F) = \varepsilon_F$.

The diagonalization is made by the following transformation

$$a_{\mathbf{k}\alpha} = \sum_{\lambda=\pm}u_{\alpha\lambda}(\mathbf{k})c_{\mathbf{k}\lambda}, \tag{9}$$

with the coefficients

$$\begin{aligned} u_{\uparrow\lambda}(\mathbf{k}) &= \sqrt{\frac{|\boldsymbol{\gamma}| + \lambda\gamma_z}{2|\boldsymbol{\gamma}|}}, \\ u_{\downarrow\lambda}(\mathbf{k}) &= \lambda\frac{\gamma_x + i\gamma_y}{\sqrt{2|\boldsymbol{\gamma}|(|\boldsymbol{\gamma}| + \lambda\gamma_z)}}, \end{aligned}$$

forming a unitary matrix $\hat{u}(\mathbf{k})$.

The zero field Green functions in the band representation are diagonal and have the following form:[9]

$$\begin{aligned} G^{(0)}_{\lambda\lambda'}(\omega_n,\mathbf{k}) &= \delta_{\lambda\lambda'}G_\lambda(\omega_n,\mathbf{k}), \\ F^{(0)}_{\lambda\lambda'}(\omega_n,\mathbf{k}) &= \delta_{\lambda\lambda'}F_\lambda(\omega_n,\mathbf{k}), \end{aligned} \tag{10}$$

where

$$\begin{aligned} G_\lambda(\omega_n,\mathbf{k}) &= -\frac{i\hbar\omega_n + \xi_\lambda}{\hbar^2\omega_n^2 + \xi_\lambda^2 + |\tilde{\Delta}_\lambda(\mathbf{k})|^2}, \\ F_\lambda(\omega_n,\mathbf{k}) &= \frac{t_\lambda(\mathbf{k})\tilde{\Delta}_\lambda(\mathbf{k})}{\hbar^2\omega_n^2 + \xi_\lambda^2 + |\tilde{\Delta}_\lambda(\mathbf{k})|^2}, \end{aligned} \tag{11}$$

and

$$t_\lambda(\mathbf{k}) = -\lambda\frac{\gamma_x(\mathbf{k}) - i\gamma_y(\mathbf{k})}{\sqrt{\gamma_x^2(\mathbf{k}) + \gamma_y^2(\mathbf{k})}}.$$

The functions $\tilde{\Delta}_\lambda(\mathbf{k})$ are the gaps in the λ-band quasiparticle spectrum in superconducting state. In the simplest model with BCS pairing interaction $v_g(\mathbf{k},\mathbf{k}') = -V_g$, the gap functions are the same in both bands: $\tilde{\Delta}_+(\mathbf{k}) = \tilde{\Delta}_-(\mathbf{k}) = \Delta$ and we deal with pure singlet pairing[18].

Transforming the Green functions using eqn. (9) into the band representation[19], we obtain

$$\begin{aligned} Tr\{\hat{v}_i(\mathbf{k})\hat{G}^{(0)}(K_+)\hat{v}_j(\mathbf{k})\hat{G}^{(0)}(K_-) \\ +\hat{v}_i(\mathbf{k})\hat{F}^{(0)}(K_+)\hat{v}^t_j(-\mathbf{k})\hat{F}^{+(0)}(K_-)\} = \\ v_{++,i}(\mathbf{k})G_+v_{++,j}(\mathbf{k})G_+ + v_{++,i}(\mathbf{k})F_+v_{++,j}(-\mathbf{k})F_+^\dagger + \\ v_{--,i}(\mathbf{k})G_-v_{--,j}(\mathbf{k})G_- + v_{--,i}(\mathbf{k})F_-v_{-+,j}(-\mathbf{k})F_+^\dagger + \\ v_{+-,i}(\mathbf{k})G_-v_{-+,j}(\mathbf{k})G_+ + v_{+-,i}(\mathbf{k})F_-v_{-+,j}(-\mathbf{k})F_+^\dagger + \\ v_{-+,i}(\mathbf{k})G_+v_{+-,j}(\mathbf{k})G_- + v_{-+,i}(\mathbf{k})F_+v_{+-,j}(-\mathbf{k})F_-^\dagger \end{aligned} \tag{12}$$

as the trace of the matrices in eqn. (6). For the brevity, we omit here the arguments of the Green functions. They are the same as in the upper two lines. The matrix velocity in the band representation is

$$\begin{aligned} \mathbf{v}_{\lambda\lambda'}(\pm\mathbf{k}) = u^\dagger_{\lambda\alpha}(\pm\mathbf{k})\mathbf{v}_{\alpha\beta}(\pm\mathbf{k})u_{\beta\lambda'}(\pm\mathbf{k}) = \\ \frac{\partial\xi_0(\pm\mathbf{k})}{\partial\mathbf{k}}\delta_{\lambda\lambda'} + \frac{\partial\gamma_l(\pm\mathbf{k})}{\partial\mathbf{k}}\tau_{l,\lambda\lambda'}(\pm\mathbf{k}), \end{aligned} \tag{13}$$

where $\hat{\boldsymbol{\tau}}(\mathbf{k}) = \hat{u}^\dagger(\mathbf{k})\hat{\boldsymbol{\sigma}}\hat{u}(\mathbf{k})$ are hermitian matrices. We have neglected[20] the difference between $\hat{u}(\mathbf{k})$ and $\hat{u}(\mathbf{k} \pm \mathbf{q}/2)$.

The explicit expressions for the $\hat{\boldsymbol{\tau}}(\mathbf{k})$ matrices are

$$\begin{aligned} \hat{\tau}_x &= \begin{pmatrix} \hat{\gamma}_x & -\frac{\gamma_x\hat{\gamma}_z + i\gamma_y}{\gamma_\perp} \\ -\frac{\gamma_x\hat{\gamma}_z - i\gamma_y}{\gamma_\perp} & -\hat{\gamma}_x \end{pmatrix}, \\ \hat{\tau}_y &= \begin{pmatrix} \hat{\gamma}_y & -\frac{\gamma_y\hat{\gamma}_z - i\gamma_x}{\gamma_\perp} \\ -\frac{\gamma_y\hat{\gamma}_z + i\gamma_x}{\gamma_\perp} & -\hat{\gamma}_y \end{pmatrix}, \\ \hat{\tau}_z &= \begin{pmatrix} \hat{\gamma}_z & \frac{\gamma_\perp}{\gamma} \\ \frac{\gamma_\perp}{\gamma} & -\hat{\gamma}_z \end{pmatrix}, \end{aligned} \tag{14}$$

where $\hat{\boldsymbol{\gamma}} = \boldsymbol{\gamma}/|\boldsymbol{\gamma}|$, $\gamma_\perp = \sqrt{\gamma_x^2 + \gamma_y^2}$. All the diagonal elements of these matrices are odd functions of the momentum components. Correspondigly, the products of the diagonal components of velocity matrices (13) are even functions of momentum. Hence, these terms in eqn. (12)

can produce only the terms proportional to even powers of the product $\mathbf{kq}$ being not responsible for the gyrotropy properties.

The off diagonal elements of $\hat{\boldsymbol{\tau}}(\mathbf{k})$ have the mixed parity. So, the dispersive terms proportional to the odd powers of the product $\mathbf{kq}$ can arise only from the part of eqn. (12) containing the off diagonal elements of $\hat{\boldsymbol{\tau}}(\mathbf{k})$ matrices. Hence, for the calculation of gyrotropy of conductivity, only the last two lines in eqn. (12) consisting of interband terms are important. They are equal to

$$\frac{\partial\gamma_l}{\partial k_i}\frac{\partial\gamma_m}{\partial k_j}\{\tau_{l,+-}\tau^*_{m,+-}[G_-G_+ - F_-F_+^\dagger] + \tau_{l,-+}\tau^*_{m,-+}[G_+G_- - F_+F_-^\dagger]\}. \quad (15)$$

Using the identity

$$\tau_{l,+-}\tau^*_{m,+-} = \tau^*_{l,-+}\tau_{m,-+} = \delta_{lm} - \hat{\gamma}_l\hat{\gamma}_m + ie_{lmn}\hat{\gamma}_n,$$

where $\hat{\gamma}_l = \gamma_l/|\boldsymbol{\gamma}|$, one can rewrite the above expression as

$$\frac{\partial\gamma_l}{\partial k_i}\frac{\partial\gamma_m}{\partial k_j}\{(\delta_{lm} - \hat{\gamma}_l\hat{\gamma}_m)[G_-G_+ + G_+G_- - F_-F_+^\dagger - F_+F_-^\dagger] + ie_{lmn}\hat{\gamma}_n[G_-G_+ - G_+G_- - F_-F_+^\dagger + F_+F_-^\dagger]\}. \quad (16)$$

Starting this point we need the explicit form of spin-orbit coupling vector $\boldsymbol{\gamma}(\mathbf{k})$. Its momentum dependence is determined by the crystal symmetry.[9,17] For the cubic group $G = O$, which describes the point symmetry of $Li_2(Pd_{1-x},Pt_x)_3B$, the simplest form compatible with the symmetry requirements is

$$\boldsymbol{\gamma}(\mathbf{k}) = \gamma_0\mathbf{k}, \quad (17)$$

where γ_0 is a constant. For the tetragonal group $G = C_{4v}$, which is relevant for $CePt_3Si$, $CeRhSi_3$ and $CeIrSi_3$, the spin-orbit coupling is given by

$$\boldsymbol{\gamma}(\mathbf{k}) = \gamma_\perp(k_y\hat{x} - k_x\hat{y}) + \gamma_\parallel k_xk_yk_z(k_x^2 - k_y^2)\hat{z}. \quad (18)$$

The gyrotropy current $\mathbf{j}_g$, which is linear with respect to the wave vector $\mathbf{q}$, originates from the last term in the eqn. (16). One can show that for the tetragonal crystal with the symmetry group $G = C_{4v}$, for the electric field lying in the basal plane the linear in the component of wave vector $\mathbf{q}$ part of conductivity is absent. In that follows we continue calculation for the metal with cubic symmetry where $\hat{\gamma} = \hat{k}\ sign\gamma_0$. We put $\hat{\gamma} = \hat{k}$ taking γ_0 as a positive constant. Thus, we obtain for gyrotropy current[21]

$$j_{gi}(\omega_n, \mathbf{q}) = ie_{ijl}\frac{e^2\gamma_0^2}{c}I_lA_j(\omega_n, \mathbf{q}), \quad (19)$$

$$I_l = \int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l \times \mathrm{T}\sum_{m=-\infty}^{\infty}[G_+(K_+)G_-(K_-) - F_+(K_+)F_-^\dagger(K_-) - G_-(K_+)G_+(K_-) + F_-(K_+)F_+^\dagger(K_-)]. \quad (20)$$

Let us find first the gyrotropy conductivity in the normal state.

III. GYROTROPY CONDUCTIVITY IN THE NORMAL STATE

Substituting the Green function in the eqn. (20) and performing summation over the Matsubara frequencies, we obtain

$$I_l = \int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l\left\{\frac{f(\xi_-(\mathbf{k}_-)) - f(\xi_+(\mathbf{k}_+))}{i\hbar\omega_n + \xi_-(\mathbf{k}_-) - \xi_+(\mathbf{k}_+)} - \frac{f(\xi_+(\mathbf{k}_-)) - f(\xi_-(\mathbf{k}_+))}{i\hbar\omega_n + \xi_+(\mathbf{k}_-) - \xi_-(\mathbf{k}_+)}\right\}. \quad (21)$$

Here $f(\xi_\pm(\mathbf{k}_\pm))$ is the Fermi distribution function and $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{q}/2$. By changing the sign of momentum $\mathbf{k} \to -\mathbf{k}$ in the first term under integral and making use that $\xi_\lambda(\mathbf{k})$ is even function of $\mathbf{k}$, we come to

$$I_l = 2\int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l\frac{[\xi_+(\mathbf{k}_+) - \xi_-(\mathbf{k}_-)][f(\xi_+(\mathbf{k}_+)) - f(\xi_-(\mathbf{k}_-))]}{(\xi_+(\mathbf{k}_+) - \xi_-(\mathbf{k}_-))^2 - (i\hbar\omega_n)^2}. \quad (22)$$

Analytical continuation of this expression from the discrete set of Matsubara frequencies into entire half-plane $\omega > 0$ is performed by the usual substitution $i\omega_n \to \omega + i/\tau$.

We shall work at frequencies smaller when the band splitting $\hbar\omega < \gamma_0k_F$ far from the resonance region $\hbar\omega \approx \gamma_0k_F$ but still in the collisionless limit $\omega\tau > 1$ where one can decompose the integrand in powers of ω^2:

$$I_l = 2\int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l[f(\xi_+(\mathbf{k}_+)) - f(\xi_-(\mathbf{k}_-))] \times\left\{\frac{1}{\xi_+(\mathbf{k}_+) - \xi_-(\mathbf{k}_-)} + \frac{(\hbar\omega)^2}{(\xi_+(\mathbf{k}_+) - \xi_-(\mathbf{k}_-))^3}\right\}, \quad (23)$$

The frequency independent term in eqn. (23) corresponds to the current density $\nu\mathbf{B}$ introduced in eqn. (4). We are interested in linear in $\mathbf{q}$ part of density of current. Expanding the integrand up to the first order in $\frac{\partial\xi_\pm}{\partial\mathbf{k}}\mathbf{q}$ one can prove by direct calculation that this term vanishes. Thus, in the normal state $\nu = 0$ as it should be in gauge invariant theory (see Section V and[14]). The frequency dependent term determines the current

$$j_i^g(\omega, \mathbf{q}) = ie_{ijl}\frac{e^2\gamma_0^2}{c}\hbar q_m(\hbar\omega)^2I_{lm}A_j(\omega, \mathbf{q}), \quad (24)$$

where

$$I_{lm} = \int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l\left[-3\frac{f(\xi_+) - f(\xi_-)}{(\xi_+ - \xi_-)^4}\left(\frac{\partial\xi_+}{\partial k_m} + \frac{\partial\xi_-}{\partial k_m}\right) + \frac{1}{(\xi_+ - \xi_-)^3}\left(\frac{\partial f(\xi_+)}{\partial\xi_+}\frac{\partial\xi_+}{\partial k_m} + \frac{\partial f(\xi_-)}{\partial\xi_-}\frac{\partial\xi_-}{\partial k_m}\right)\right]. \quad (25)$$

After substitution of the Fourier component of the vector potential by the Fourier component of an electric field $\mathbf{A} = c\mathbf{E}/i\omega$, we obtain

$$j_i^g(\omega, \mathbf{q}) = e_{ijl}e^2\hbar^3\gamma_0^2\omega q_mI_{lm}E_j(\omega, \mathbf{q}). \quad (26)$$

The integral I_{lm} given by eqn. (25) consists of two different contributions. The first part of it is determined by the difference of the Fermi distribution function for the quasiparticles in two bands, another one originates from the derivatives of these functions. Performing integration over momentum space for the spherical Fermi surfaces in the limit $\gamma_0 k_F \ll \varepsilon_F$, we obtain

$$j_i^g(\omega,\mathbf{q}) = e_{ijl}\left(\frac{1}{8}-\frac{1}{24}\right)\frac{e^2\omega}{\pi^2\gamma_0 k_F}q_l E_j(\omega,\mathbf{q}) = \\ = e_{ijl}\frac{e^2\omega}{12\pi^2\gamma_0 k_F}q_l E_j(\omega,\mathbf{q}). \quad (27)$$

Here, in the first line, we mark out the contributions from the two parts of the integral (25). The corresponding gyrotropy conductivity is

$$\lambda = i\frac{e^2\omega}{12\pi^2\gamma_0 k_F}. \quad (28)$$

IV. GYROTROPY CONDUCTIVITY IN THE SUPERCONDUCTING STATE

To find the gyrotropy conductivity in the superconducting phase with the cubic symmetry, one needs to perform summation and integration in the eqn. (20) using G and F in the superconducting phase. The integral in eqn. (20) consists of two terms

$$I_l = \int \frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l[J_{+-}(\mathbf{k},\omega) - J_{-+}(\mathbf{k},\omega)], \quad (29)$$

where

$$J_{+-}(\mathbf{k},\omega) = T\sum_{m=-\infty}^{\infty}[G_+(K_+)G_-(K_-) - F_+(K_+)F_-^\dagger(K_-)], \quad (30)$$

and

$$J_{-+}(\mathbf{k},\omega) = T\sum_{m=-\infty}^{\infty}[G_-(K_+)G_+(K_-) - F_-(K_+)F_+^\dagger(K_-)]. \quad (31)$$

Transforming the summation into an equivalent contour integration[16], eqn. (30) can be written as

$$\begin{aligned} J_{+-}(\mathbf{k},\omega) &= \frac{\hbar}{4\pi i}\oint d\omega' \tanh\frac{\omega'}{2T} \\ &\times \{[G_+^R(\omega',\mathbf{k}_+) - G_+^A(\omega',\mathbf{k}_+)]G_-^A(\omega'-\omega,\mathbf{k}_-) \\ &+[G_-^R(\omega',\mathbf{k}_-) - G_-^A(\omega',\mathbf{k}_-)]G_+^R(\omega'+\omega,\mathbf{k}_+) \\ &-[F_+^R(\omega',\mathbf{k}_+) - F_+^A(\omega',\mathbf{k}_+)]F_-^A(\omega'-\omega,\mathbf{k}_-) \\ &-[F_-^R(\omega',\mathbf{k}_-) - F_-^A(\omega',\mathbf{k}_-)]F_+^R(\omega'+\omega,\mathbf{k}_+)\}. \end{aligned} \quad (32)$$

Here, the Green functions are

$$G_\lambda^{R,A}(\omega,\mathbf{k}) = \frac{u_\lambda^2(\mathbf{k})}{\hbar\omega - \epsilon_\lambda(\mathbf{k}) \pm i\delta} + \frac{v_\lambda^2(\mathbf{k})}{\hbar\omega + \epsilon_\lambda(\mathbf{k}) \pm i\delta}, \quad (33)$$

and

$$F_\lambda^{R,A}(\omega,\mathbf{k}) = \frac{t_\lambda(\mathbf{k})\Delta}{2\epsilon_\lambda(\mathbf{k})}\left[\frac{1}{\hbar\omega + \epsilon_\lambda(\mathbf{k}) \pm i\delta} - \frac{1}{\hbar\omega - \epsilon_\lambda(\mathbf{k}) \pm i\delta}\right], \quad (34)$$

where

$$\left.\begin{matrix} u_\lambda^2(\mathbf{k}) \\ v_\lambda^2(\mathbf{k}) \end{matrix}\right\} = \frac{1}{2}\left(1 \pm \frac{\xi_\lambda(\mathbf{k})}{\epsilon_\lambda(\mathbf{k})}\right), \quad (35)$$

$$\epsilon_\lambda^2(\mathbf{k}) = \xi_\lambda^2(\mathbf{k}) + \Delta^2, \quad (36)$$

and $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{q}/2$. Taking into account

$$G_\lambda^R - G_\lambda^A = -2\pi i[u_\lambda^2\delta(\omega - \epsilon_\lambda) + v_\lambda^2\delta(\omega + \epsilon_\lambda)], \quad (37)$$

$$F_\lambda^R - F_\lambda^A = \frac{\pi i t_\lambda \Delta}{\epsilon_\lambda}[\delta(\omega - \epsilon_\lambda) - \delta(\omega + \epsilon_\lambda)], \quad (38)$$

after integration with respect to ω', we can rewrite eqn. (32) as:

$$\begin{aligned} J_{+-}(\mathbf{k},\omega) = -\frac{1}{2}\Bigg[&\left(\tanh\frac{\epsilon_+(\mathbf{k}_+)}{2T} - \tanh\frac{\epsilon_-(\mathbf{k}_-)}{2T}\right)\left(\frac{u_+^2(\mathbf{k}_+)u_-^2(\mathbf{k}_-)}{\epsilon_+(\mathbf{k}_+) - \epsilon_-(\mathbf{k}_-) - \omega} + \frac{v_+^2(\mathbf{k}_+)v_-^2(\mathbf{k}_-)}{\epsilon_+(\mathbf{k}_+) - \epsilon_-(\mathbf{k}_-) + \omega}\right) \\ &+\left(\tanh\frac{\epsilon_+(\mathbf{k}_+)}{2T} + \tanh\frac{\epsilon_-(\mathbf{k}_-)}{2T}\right)\left(\frac{u_+^2(\mathbf{k}_+)v_-^2(\mathbf{k}_-)}{\epsilon_+(\mathbf{k}_+) + \epsilon_-(\mathbf{k}_-) - \omega} + \frac{v_+^2(\mathbf{k}_+)u_-^2(\mathbf{k}_-)}{\epsilon_+(\mathbf{k}_+) + \epsilon_-(\mathbf{k}_-) + \omega}\right)\Bigg] \\ -\frac{1}{2}\frac{\Delta^2}{4\epsilon_+(\mathbf{k}_+)\epsilon_-(\mathbf{k}_-)}\Bigg[&-\left(\tanh\frac{\epsilon_+(\mathbf{k}_+)}{2T} - \tanh\frac{\epsilon_-(\mathbf{k}_-)}{2T}\right)\left(\frac{1}{\epsilon_+(\mathbf{k}_+) - \epsilon_-(\mathbf{k}_-) - \omega} + \frac{1}{\epsilon_+(\mathbf{k}_+) - \epsilon_-(\mathbf{k}_-) + \omega}\right) \\ &+\left(\tanh\frac{\epsilon_+(\mathbf{k}_+)}{2T} + \tanh\frac{\epsilon_-(\mathbf{k}_-)}{2T}\right)\left(\frac{1}{\epsilon_+(\mathbf{k}_+) + \epsilon_-(\mathbf{k}_-) - \omega} + \frac{1}{\epsilon_+(\mathbf{k}_+) + \epsilon_-(\mathbf{k}_-) + \omega}\right)\Bigg]. \end{aligned} \quad (39)$$

Here we have ignored the shifts in the arguments of the phase factors: $t_\lambda(\mathbf{k}\pm\mathbf{q/2})\approx t_\lambda(\mathbf{k})$ leading to the small corrections of the order of $\gamma_0 k_F/\varepsilon_F$ to the main terms.

For the second term under integral in the eqn. (29) we have

$$J_{-+}(\mathbf{k},\omega)=J_{+-}(-\mathbf{k},-\omega). \tag{40}$$

Hence, the integral (29) can be rewritten as

$$I_l=\int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l[J_{+-}(\mathbf{k},\omega)+J_{+-}(\mathbf{k},-\omega)]. \tag{41}$$

It means that we should work with the doubled even part of eqn. (39). Expanding the integrand in powers of ω after long but straightforward calculations we come to the following formula

$$\begin{aligned}I_l=&\,2\int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l[n(\xi_+(\mathbf{k}_+))-n(\xi_-(\mathbf{k}_-))]\\&\times\left\{\frac{1}{\xi_+(\mathbf{k}_+)-\xi_-(\mathbf{k}_-)}+\frac{(\hbar\omega)^2}{(\xi_+(\mathbf{k}_+)-\xi_-(\mathbf{k}_-))^3}\right\}\\&-2\Delta^2\int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l\frac{(\hbar\omega)^2}{(\xi_+(\mathbf{k}_+)+\xi_-(\mathbf{k}_-))(\xi_+(\mathbf{k}_+)-\xi_-(\mathbf{k}_-))^3}\\&\times\left(\frac{\tanh\frac{\epsilon_+(\mathbf{k}_+)}{2T}}{\epsilon_+(\mathbf{k}_+)}-\frac{\tanh\frac{\epsilon_-(\mathbf{k}_-)}{2T}}{\epsilon_-(\mathbf{k}_-)}\right),\end{aligned} \tag{42}$$

where

$$n(\xi)=\frac{1}{2}\left(1-\frac{\xi}{\epsilon}\tanh\frac{\epsilon}{2T}\right) \tag{43}$$

is the distribution function of electrons over energies. At $\Delta/\gamma_0 k_F\ll 1$ the second integral is obviously much smaller than the first one. So, we come to the expression

$$\begin{aligned}I_l\cong&\,2\int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l[n(\xi_+(\mathbf{k}_+))-n(\xi_-(\mathbf{k}_-))]\\&\times\left\{\frac{1}{\xi_+(\mathbf{k}_+)-\xi_-(\mathbf{k}_-)}+\frac{(\hbar\omega)^2}{(\xi_+(\mathbf{k}_+)-\xi_-(\mathbf{k}_-))^3}\right\}\end{aligned} \tag{44}$$

which has the same form as the corresponding formula for the normal state (23).

Expanding the integrand up to the first order in $\frac{\partial\xi_\pm}{\partial\mathbf{k}}\mathbf{q}$ we obtain for the current given by eqn. (19):

$$j_i^g(\omega,\mathbf{q})=ie_{ijl}\frac{e^2\gamma_0^2}{c}\hbar q_m[I^1_{lm}+(\hbar\omega)^2I^3_{lm}]A_j(\omega,\mathbf{q}), \tag{45}$$

$$\begin{aligned}I^1_{lm}=&\int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l\left[-\frac{n(\xi_+)-n(\xi_-)}{(\xi_+-\xi_-)^2}\left(\frac{\partial\xi_+}{\partial k_m}+\frac{\partial\xi_-}{\partial k_m}\right)\right.\\&\left.+\frac{1}{\xi_+-\xi_-}\left(\frac{\partial n(\xi_+)}{\partial\xi_+}\frac{\partial\xi_+}{\partial k_m}+\frac{\partial n(\xi_-)}{\partial\xi_-}\frac{\partial\xi_-}{\partial k_m}\right)\right],\end{aligned} \tag{46}$$

$$\begin{aligned}I^3_{lm}=&\int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l\left[-3\frac{n(\xi_+)-n(\xi_-)}{(\xi_+-\xi_-)^4}\left(\frac{\partial\xi_+}{\partial k_m}+\frac{\partial\xi_-}{\partial k_m}\right)\right.\\&\left.+\frac{1}{(\xi_+-\xi_-)^3}\left(\frac{\partial n(\xi_+)}{\partial\xi_+}\frac{\partial\xi_+}{\partial k_m}+\frac{\partial n(\xi_-)}{\partial\xi_-}\frac{\partial\xi_-}{\partial k_m}\right)\right].\end{aligned} \tag{47}$$

As in the normal state, the gyrotropic current consists of two different contributions. The first part of it is determined by the quasiparticles filling up the states in between two Fermi surfaces. Another part originates from the derivatives of electron distribution functions in two bands. The first contribution is not changed in the superconducting state, at $\Delta<<\gamma_0 k_F$. But the second contribution in the superconducting state is suppressed in comparison with its normal value due to the gap in the quasiparticle spectrum. Neglecting the terms of the order of $\Delta^2/\gamma_0^2k_F^2$ one can substitute under integrals the derivatives of the particle distribution functions $\partial n(\xi_\pm)/\partial\xi_\pm$ by the derivatives of the quasiparticles distribution function $\partial f(\epsilon_\pm)/\partial\epsilon_\pm$, where $f(\epsilon)=(1-\tanh\frac{\epsilon}{2T})/2$. This leads to the temperature dependence of gyrotropy coefficient $\lambda(T)$ determined by the integral I^3_{lm}

$$\lambda(T)=i\frac{e^2\omega}{8\pi^2\gamma_0k_F}(1-\frac{1}{3}Y(T)). \tag{48}$$

Here $Y(T)=\frac{1}{4T}\int\frac{d\xi}{\cosh^2\epsilon/2T}$ is the Yosida function. It is useful to compare this formula with the corresponding normal state equation (27).

Besides of this term there is another part of the current originating of the integral I^1_{lm}. It yields the coefficient

$$\nu(T)=\frac{e^2\gamma_0k_F}{6\pi^2\hbar^2c}(1-Y(T)) \tag{49}$$

such that the total gyrotropy coefficient $\Lambda(T)$ in the superconducting state is

$$\Lambda(T)=i\frac{e^2}{4\pi^2\hbar}\left[\frac{\hbar\omega}{2\gamma_0k_F}\left(1-\frac{1}{3}Y(T)\right)-\frac{2\gamma_0k_F}{3\hbar\omega}\left(1-Y(T)\right)\right] \tag{50}$$

To find a relationship of the gyrotropy conductivity with observable optical properties one has to develop electrodynamic theory of noncentrosymmetric metals.

V. OPTICAL PROPERTIES OF NONCENTROSYMMETRIC METAL

A. Dispersion law

To derive the light dispersion law we start from the Maxwell equations

$$\mathrm{rot}\mathbf{B}=\frac{4\pi}{c}\mathbf{j}\,, \tag{51}$$

$$\mathrm{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \tag{52}$$

supplied by the density of current expression

$$\mathbf{j}=\frac{\epsilon}{4\pi}\frac{\partial\mathbf{E}}{\partial t}+\sigma\mathbf{E}+\lambda\mathrm{rot}\mathbf{E}. \tag{53}$$

The first term here corresponds to the dispersionless part of the displacement current. The second one is the conductivity current written at infrared frequency region

$\omega > v_F/\delta,\ \omega\tau > 1$, where the current is locally related with an electric field, δ is the skin penetration depth. The last one is the gyrotropy current

$$\mathbf{j}_g = \lambda \mathrm{rot}\mathbf{E}. \tag{54}$$

As before we discuss the metal with the cubic point symmetry.

Eliminating the magnetic induction, we obtain

$$\nabla^2\mathbf{E} = \frac{\epsilon}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} + \frac{4\pi\sigma}{c^2}\frac{\partial\mathbf{E}}{\partial t} + \frac{4\pi\lambda}{c^2}\frac{\partial \mathrm{rot}\mathbf{E}}{\partial t}. \tag{55}$$

Taking solution for the circularly polarized wave

$$\mathbf{E} = (\hat{x} \pm i\hat{y})E_0 e^{i(\mathbf{kr}-\omega t)} \tag{56}$$

we come to the dispersion relation

$$k^2 = \frac{\epsilon\omega^2}{c^2} + \frac{4\pi i\sigma\omega}{c^2} \pm \frac{4\pi i\lambda\omega k}{c^2}. \tag{57}$$

It is worth to be noted that for a media with time reversal breaking one has to substitute here $\sigma \to \sigma_\pm = \sigma_{xx} \pm i\sigma_{xy}$, where σ_{xy} is the Hall conductivity.

In neglect the gyrotropy term the complex index of refraction

$$N = \frac{ck}{\omega} = n + i\kappa$$

is expressed through the diagonal part of complex conductivity $\sigma = \sigma' + i\sigma''$ by means of the usual relations

$$n^2 - \kappa^2 = \epsilon - \frac{4\pi\sigma''}{\omega}, \qquad 2n\kappa = \frac{4\pi\sigma'}{\omega}.$$

The gyrotropy term leads to the difference in the refraction indices of clock wise and counter clock wise polarized light. In the first order in respect to $\lambda = \lambda' + i\lambda''$ the refraction index is

$$N^\pm = n + i\kappa \pm \frac{2\pi i\lambda}{c}. \tag{58}$$

Hence, the differences in the real and imaginary parts of the refraction indices of circularly polarized lights with the opposite polarization are

$$\Delta n = n_+ - n_- = -\frac{4\pi\lambda''}{c}, \tag{59}$$

$$\Delta\kappa = \kappa_+ - \kappa_- = \frac{4\pi\lambda'}{c}. \tag{60}$$

In the superconducting state the gyrotropy current (54) has more general form given by eqn. (4). Hence, we should use the more general formula for the current

$$\mathbf{j} = \frac{\epsilon}{4\pi}\frac{\partial\mathbf{E}}{\partial t} + \sigma\mathbf{E} + \lambda\mathrm{rot}\mathbf{E} + \nu\mathbf{B}. \tag{61}$$

Then repeating all the calculations we come to the same results (57)-(60) modified by the substitution

$$\lambda \ \to\ \Lambda = \lambda - \frac{ic\nu}{\omega}. \tag{62}$$

We remind that the superconducting state current density given by eqn. (61) is worth to use at the high frequencies $\omega > v_F/\delta$ where the inequality $\hbar\omega >> \Delta$ is certainly valid. Here, Δ is superconducting energy gap. In the low frequency limit $\hbar\omega < \Delta$ one should also take into account the London density of current $\mathbf{j}_L = -(c/4\pi\delta_L^2)(\mathbf{A} - \hbar c\nabla\varphi/2e)$. The interplay between the London current $\mathbf{j}_L$ and the Drude current $\mathbf{j}_D = \sigma(\omega)\mathbf{E}$ is discussed in the textbook.[16]

B. Magnetic moment

The magnetization in gyrotropic media is

$$\mathbf{M} = \frac{1}{2c}\lambda\mathbf{E}, \tag{63}$$

such that rotation of the magnetization is equal to one half of gyrotropy part of the current density

$$\frac{1}{2}\,\mathbf{j}_g = c\ \mathrm{rot}\mathbf{M}. \tag{64}$$

The relationship between the density of gyrotropy current and the magnetization is a general property of noncentrosymmetric materials (see also[13]). Both of them can be obtained from the gyrotropy term in action

$$-\frac{1}{2c}\int dt d^3\mathbf{r}(\lambda\mathbf{E})\mathbf{B}.$$

By variation of action in respect of $-\mathbf{B}$ and $-\mathbf{A}/c$, taking into account that λ is an odd function of derivative $\partial/\partial t$ and making use the definitions $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$ and $\mathbf{B} = \mathrm{rot}\mathbf{A}$, we come to $\mathbf{M}$ given by eqn.(63) and $\mathbf{j}_g$ given by eqn.(54) correspondingly.

All these considerations are valid both for the normal and as well for the superconducting state. However, in the latter case the gyrotropy action

$$S_g = -\frac{1}{2c}\int dt d^3\mathbf{r}\left\{(\lambda\mathbf{E})\mathbf{B} + \nu\left[\mathbf{A} - \frac{\hbar c}{2e}\nabla\varphi\right]\mathbf{B}\right\}. \tag{65}$$

contains one extra term which is absent in the normal state due to the gauge invariance. The corresponding expressions for the magnetic moment and gyrotropy current are

$$\mathbf{M} = \frac{1}{2c}\lambda\mathbf{E} + \frac{1}{2c}\nu\left[\mathbf{A} - \frac{\hbar c}{2e}\nabla\varphi\right], \tag{66}$$

$$\mathbf{j}_g = \lambda\ \mathrm{rot}\mathbf{E} + \nu\mathbf{B}. \tag{67}$$

C. Boundary conditions

To consider the problem of light reflection normally incident to the flat surface of noncentrosymmetric metal we need to find the relations between the wave amplitude propagating inside ($z>0$) the material

$$\mathbf{E}^{in}=\mathbf{E}_0 e^{i\omega(Nz/c-t)} \tag{68}$$

and the amplitudes of incident and reflected waves outside it

$$\mathbf{E}^{out}=\mathbf{E}_1 e^{i\omega(z/c-t)}+\mathbf{E}_2 e^{-i\omega(z/c+t)}. \tag{69}$$

We have

$$\mathbf{E}^{in}_{z=o}=\mathbf{E}^{out}_{z=0}$$

that is

$$\mathbf{E}_0=\mathbf{E}_1+\mathbf{E}_2. \tag{70}$$

At the same time from the difference of the Maxwell equations (52) inside and outside of material we obtain

$$(\mathrm{rot}\mathbf{E}^{in}-\mathrm{rot}\mathbf{E}^{out})_{z=0}=-\frac{1}{c}\frac{\partial}{\partial t}(\mathbf{B}^{in}-\mathbf{B}^{out})_{z=0}. \tag{71}$$

The difference of the magnetic inductions at the boundary is given by the jump of magnetzation

$$(\mathbf{B}^{in}-\mathbf{B}^{out})_{z=0}=4\pi\mathbf{M}_{z=0}. \tag{72}$$

In the stationary magnetic field parallel to the surface of the metal $\mathbf{H}^{out}=H_x\hat{x}$ this equation yields

$$B_x^{int}(z=0)=H_x, \qquad B_y^{int}(z=0)=\frac{2\pi\kappa}{c}A_y^{int}(z=0). \tag{73}$$

In the normal state where $\kappa=0$ the boundary conditions add nothing special to the centrosymmetric case. In the superconducting state the solution of the London equations supplied by these boundary conditions results in quite unusual helical field distribution found in the paper.[14]

For the light incident to the metallic surface using (71), (72), and (66) we obtain

$$(\mathrm{rot}\mathbf{E}^{in}-\mathrm{rot}\mathbf{E}^{out})_{z=0}=-\frac{4\pi}{2c^2}\left(\lambda\frac{\partial}{\partial t}-c\nu\right)\mathbf{E}^{in}_{z=0} \tag{74}$$

Substituting here eqns. (68), (69) we come to

$$\hat{z}\times(N\mathbf{E}_0-\mathbf{E}_1+\mathbf{E}_2)=\frac{2\pi}{c}\Lambda\mathbf{E}_0 \tag{75}$$

For the combinations $E^{\pm}=E_x\pm iE_y$ of the electric field component this relation can be rewritten as

$$\left(N^{\pm}\pm\frac{2\pi i\Lambda}{c}\right)E_0^{\pm}-E_1^{\pm}+E_2^{\pm}=0 \tag{76}$$

D. Reflection coefficient and the Kerr effect

The equations (70) and (76) allow express the amplitudes of reflected wave through the amplitude of the incident wave. We have for reflection coefficient

$$R^{\pm}=\frac{E_2^{\pm}}{E_1^{\pm}}=\frac{1-N^{\pm}\mp\frac{2\pi i\Lambda}{c}}{1+N^{\pm}\pm\frac{2\pi i\Lambda}{c}}, \tag{77}$$

where the refraction index is

$$N_{\pm}=n+i\kappa\pm\frac{2\pi i\Lambda}{c}. \tag{78}$$

Now one can rewrite eqn. (77) in more habitual form

$$R^{\pm}=\frac{1-\tilde{N}^{\pm}}{1+\tilde{N}^{\pm}}, \tag{79}$$

where an effective refraction index is

$$\tilde{N}_{\pm}=n+i\kappa\pm\frac{4\pi i\Lambda}{c}, \tag{80}$$

and the effective differences in the real and imaginary parts of the refraction indices of circularly polarized lights with the opposite polarization are

$$\Delta\tilde{n}=\tilde{n}_+-\tilde{n}_-=-\frac{8\pi\Lambda''}{c}, \tag{81}$$

$$\Delta\tilde{\kappa}=\tilde{\kappa}_+-\tilde{\kappa}_-=\frac{8\pi\Lambda'}{c}. \tag{82}$$

Making use these definitions we can apply the standard procedure[22] to calculate the Kerr rotation for linearly polarized light normally incident from vacuum to the flat boundary of a medium. The light is reflected as elliptically polarized with the major axis rotated relative to the incident polarization by an amount

$$\theta=\frac{(1-n^2+\kappa^2)\Delta\tilde{\kappa}+2n\kappa\Delta\tilde{n}}{(1-n^2+\kappa^2)^2+(2n\kappa)^2}. \tag{83}$$

VI. THE KERR ROTATION

To find the Kerr rotation in the normal state let us substitute the eqn. (28) in eqns. (81), (82). We find $\Delta\tilde{\kappa}=0$ and $\Delta\tilde{n}$ expresses through ratio of the light frequency to the band splitting $2\gamma_0 k_F$ as

$$\Delta\tilde{n}=-\frac{2\alpha}{3\pi}\frac{\hbar\omega}{\gamma_0 k_F}. \tag{84}$$

Here, $\alpha=e^2/\hbar c$ is the fine structure constant.

We limit ourselves by the frequencies not exceeding the band splitting $\gamma_0 k_F$. Although the latter is not known for many noncentrosymmetric materials, one can expect it is about hundred Kelvin or in the frequency units

$\sim\ 10^{13} rad/sec$.[23] As an example we consider the situation when the frequency of light is of the order of this value and larger than the quasiparticles scattering rate (clean limit): $1 << \omega\tau < \omega_p\tau$, where $\omega_p = \sqrt{4\pi ne^2/m^*}$ is the plasma frequency. In this frequency region the real and imaginary part of conductivity are $\sigma' \approx \omega_p^2/4\pi\omega^2\tau$ and $\sigma'' \approx \omega_p^2/4\pi\omega$. Then, one can find $2n\kappa \approx \omega_p^2/\omega^3\tau$ and $\kappa^2 - n^2 \approx \omega_p^2/\omega^2$. Thus, for the Kerr angle we obtain

$$\theta \approx -\frac{2\alpha}{3\pi}\frac{\hbar\omega^2}{\gamma_0 k_F \omega_p^2 \tau}. \tag{85}$$

So, the Kerr angle in noncentrosymmetric metals can have measurable magnitude, in particular if we compare it with the Kerr angle of the order of 6×10^{-8} rad measured in the superconducting Sr_2RuO_4 by the Stanford group.[24]

For $\Delta\tilde{n}$ in the superconducting state we obtain

$$\Delta\tilde{n} = -\frac{2\alpha}{\pi}\left[\frac{\hbar\omega}{2\gamma_0 k_F}\left(1-\frac{1}{3}Y(T)\right) - \frac{2\gamma_0 k_F}{3\hbar\omega}\left(1-Y(T)\right)\right]. \tag{86}$$

Finally, for the Kerr angle in the same frequency interval as for the normal state we have

$$\theta = -\frac{2\alpha\omega}{\pi\omega_p^2\tau}\left[\frac{\hbar\omega}{2\gamma_0 k_F}\left(1-\frac{1}{3}Y(T)\right) - \frac{2\gamma_0 k_F}{3\hbar\omega}\left(1-Y(T)\right)\right]. \tag{87}$$

VII. CONCLUSION

We have presented here the derivation of the current response to the electromagnetic field with finite frequency and wave vector in noncentrosymmetric metal. The obtained general formula valid both in the normal and in the superconducting state was applied to the calculation of observable physical properties in the frequency interval smaller than the band splitting $\hbar\omega < \gamma_0 k_F$. The latter in its turn was supposed to be smaller than the Fermi energy $\gamma_0 k_F < \varepsilon_F$. The calculations was performed in the clean case $\omega\tau > 1$, that is, in particular, important to neglect the vortex corrections. We did not discuss the anomalous skin effect assuming that the wave length does not exceed the skin penetration depth $\delta > v_F/\omega$. In the normal state the current contains the gyrotropic part which is odd function of the wave vector and the frequency. It presents a sort of displacement current originating of band splitting in noncentrosymmetric metal. In the superconducting state there is an additional part of the gyrotropy current proportional to magnetic field. The temperature dependence of gyrotropy conductivity in the superconducting state was found. As an example the Kerr rotation for the polarized light reflected from the surface of noncentrosymmetric metal with cubic symmetry is calculated.

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